

Improved estimates of the Saturn V velocity and its ability to place the stated payload into lunar orbit

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Abstract

In the paper [1] an assessment was carried out of the Saturn V booster and the Apollo 11 spacecraft velocity at first stage separation point based on the motion picture film footage and still photographs. The estimate obtained was that the velocity achieved was significantly (800-1100m/s) lower than that required to satisfy the flight plan for propelling this mission to the surface of the Moon. Further study concludes that no more than 28 tons, including the Apollo 11 craft, out of 46 tons as stated by NASA could have been placed into lunar orbit.

Introduction

Over the past year, after writing the first paper, some comments have been received and clarifications requested. As a result, a few models have been adjusted, and so the estimates are re-presented here simply and clearly.

In addition, it became clear that concepts such as supersonic flows, shocks and shock waves even for readers with background in natural science and with technical education can appear rather vague. These concepts were introduced in a rather formal and mathematical way. Therefore for this paper the physical meaning of the conceptual apparatus used in the original assessment is presented.

Contents of the paper [1]

In [1] a description of the film footage [5] was provided. The sequence with a duration of 30 seconds (according to the timer) contained 726 frames, which corresponds with a filming speed of 24 frames per second. The frames were numbered consecutively from beginning to end.

The first 165 frames (~ 7 seconds) record the flight of the Saturn V rocket with the main engines of the first stage firing. Thereafter, the smoke exhaust of the engines dramatically contracts. Formation of new exhaust corresponding to the second stage solid fuel rockets firing (for fuel ullage) is registered on frame 179. Significant expansion and subsequent hiding of the rocket exhaust cloud occurs due to firing of the first stage solid propellant retro-rockets is recorded on frame 189.

After frame 210 the exhaust cloud of the expended solid propellants begins to reveal the head section of the rocket, the exhaust cloud is now braking and starting to lag.

Around frame 266 the glow to the aft of the second stage grows brighter. This is three seconds after the start of separation, and the second stage motors reach nominal thrust. This occurrence is in accord with the published flight sequence.

Specified time markers spanning frames 179 to 266 – covering the firing of ullage and retro rockets, reaching the nominal thrust from the second stage liquid propellant engines – in [1] were used to confirm the correctness of relying on the filming speed of 24 fps for time measurement.

During further movement of the lead part and first stage we noted inadequate lag of the first stage from the second stage. The distance between the stages over the 13 seconds following the separation extended approximately three times the leading part, i.e. approx 180m. However, there was no emphasis on contradicting the data regarding the flight sequencing and the observed operation of the second stage engines in the original paper. This has been addressed.

The main purpose of the study was the evaluation of the velocity of the Saturn V at separation. It was regarded as insufficient. Abstract:

A frame-by-frame examination of the motion picture film footage of the first stage separation of the Apollo 11 Saturn V rocket was made. The velocity achieved at the separation point was found to be significantly (800-1100m/s) lower than that required to satisfy the stated flight plan. This finding implies that the declared payload needed for a return lunar mission could not have been propelled to the Moon.

Selected data about shocks (shock waves)

Imagine that a closely fitting piston is moving in a long cylinder and pushing air ahead of itself.

At low speed, about 70 m/s, compressed air in front of a piston is negligible. The velocity of molecules is much higher than the piston velocity. Momentum transferred by the piston to the molecules is transmitted to an increasing number of air molecules. The speed of this signal transfer (disturbance) is the speed of sound.

With increasing velocity of a piston to 70-100 m/s air compression in front of the piston is already difficult to ignore. Molecules still have a velocity greater than the speed of the piston, but this excess is not large. Air begins to show significant flexibility. A further increase in speed leads to a rapid increase in resistance. A significant layer of air ahead of the piston will "know" that the piston is moving. After one second, one way or another, the molecules at a distance of 330 meters from the piston will start to move.

When the piston exceeds the speed of sound a qualitatively new phenomenon appears – a shock. The piston is now moving faster than molecules can send traffic between each other.

A "plug" of compressed air forms between the piston and the undisturbed air. In front of the shock the air does not "know" that momentarily it will be moved. After the shock, the air moves at the speed of piston. But how do the front layers "know" that they are being pushed from behind? The air in the "plug" is not only compressed but also warmed up. The speed of sound in hot air rises in proportion to the square root of the temperature. For front layers to "know" for sure the speed with which they should move, a prerequisite is a pretty good equation of speed of sound in a "plug" to the speed at which piston moves. The described shock is called a "normal shock" – the flow is perpendicular to the front of the shock.

Continuous change in the physical parameters of the gas (pressure, density, temperature, speed) occurs at very short distances, compared to the mean free path of molecules. Table 1 shows the calculated depth of a shock, divided by the mean free path λ of molecules in undisturbed gas depending on the ratio of the pressure behind the shock to the pressure of undisturbed gas [2].

Table 1. Depth of the shock

p_2/p_1	2	5	10	50	100	1000
d/λ	6.5	3.85	2.9	1.9	1.8	1.7

Such an abrupt change of parameters in most cases in practice is very well mathematically described as an abrupt discontinuity.

Before a shock, intact gas has no directional velocity, it has the density, pressure and temperature of the undisturbed air. After this surface of the shock front, ALL molecules acquire the velocity of directed motion. Density, pressure, and temperature rise.

In practice fast flying bodies such as a shell or a bullet act in place of a piston. It could be compressed gas emanating from a cylinder, a jet engine, a rifle or a gun barrel, resulting in an explosion. A normal shock, which is formed in such cases, has another name – a shock wave.

A shock wave is not usually visible. But it can be made visible in experiments, for example, by using so-called shadow and interference techniques (Image 1).



Image 1a

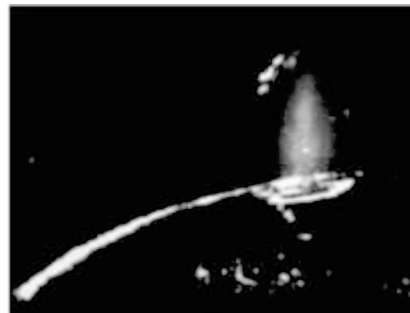


Image 1b

Image 1. Interference a) [3] and shadow b) [4] pictures of shock waves.

In the case of a nuclear explosion the shock front coincides with the boundary of the expanding luminous region.

The shock front in some cases may also group airborne smoke particles and become visible as a result. Let us examine this case because it is important.

A) If a shock wave hits the air with smoke particles then air molecules begin to move at the speed of the shock. They also carry along fine aerosol particles that make up the smoke cloud. Later the captured shock wave molecules are displaced rearwards from the front, yielding space to the new ones. But speed of this flow is only slightly less than the speed of the shock. Therefore, smoke particles are pinned to the front, and housed together with the co-current flow of compressed air at a short distance behind the shock wave. Accordingly they decorate it.

B) In another example of a typical explosion, the smoke particles arise in the explosion itself and fly away with the explosion gases. Only massive bodies with large inertia can outrun the slowing explosion shock. Nanometer-sized smoke particles fly at virtually the speed of the gas flow. Conversely, a shock wave can detach from the smoke and progress due to the capture of more and more new layers of undisturbed air and the lagging co-current flow containing smoke particles. In this case, if the speed is determined based on the moving smoke, the estimate is conservative. The speed of the shock is slightly higher.

The above facts are sufficient in order to understand one method of velocity calculation.

Speed estimate based on a diameter of the smoke cloud front

At the booster Saturn V staging, the solid fuel retro-rockets are fired emitting a cloud of smoke in a forward direction. This cloud outruns the rocket and hides it for about 0.8 sec (Image 2 below).

Consequently, the speed of smoke in this case is not lower than the speed of the rocket. The rocket speed must be supersonic. Therefore, the smoke cloud is spreading forward along the rocket course not like a usual cloud, but following the shock wave. The speed of the shock wave is even higher.

A moving shock wave necessarily evolves into movement at its own speed along with all the air molecules. But, lagging behind the shock the molecules are slowing down; their acquired kinetic energy dissipates and transforms into heat. The very area covered by the moving air expands in a radial direction. And momentum gained by molecules in the shock is redistributed into this ever-increasing mass of air.

Both energy and momentum gained by the molecules of undisturbed air when crossing the shock front originate from the energy and momentum of the one ton of the solid fuel exhaust gases, which are ejected from the nozzles of the eight retro-rockets.

In [1] the energy balance of the solid propellant retro-rocket gases transferred at the front of the smoke cloud to the air was evaluated. The estimate was not evident because of the need to introduce into consideration additional factors: temperature, heat capacity, and the radiating energy by the front.

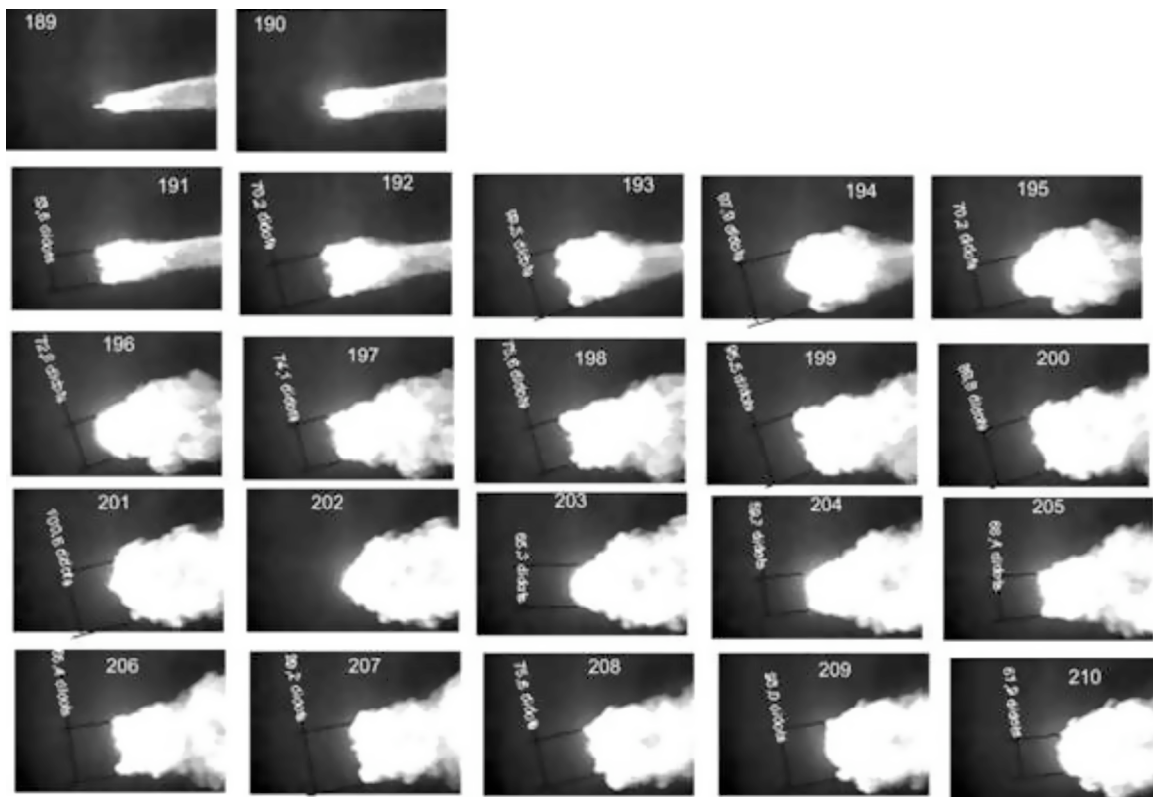


Image 2. Diameters of the aerosol cloud front – frames 189-210. The average radius of the cloud front, starting with frame 192, 61 ± 3 m [1].

A far more intuitive assessment is to use the law of conservation of linear momentum. The total momentum transferred from the retro-rockets to the Saturn V booster is the product of the engines' thrust $F = 39 \times 8 = 312$ ton-force and firing time $\tau = 0.66$ sec. It equals the momentum of gases emitted from the retro-rocket nozzles in the booster reference frame. Acceleration of air at the shock front ahead of the nose cone occurs solely due to this part of the momentum. Part of the gases momentum, equal to the product of their masses and the rocket speed in the static air reference frame, is not transferred at the shock front, because it is a condition that the gases themselves will reach the shock front. This part of the momentum stays with the gases pushing the shock front.

The momentum transferred to the front with the encountered air molecules, is $\pi R^2 \rho \tau_1 \cdot V^2$. Here V should be the speed of the shock. A shock wave with velocity V over a period of time travels $V \cdot \tau_1$. If the leading front radius is R and air density ρ , the mass involved in the movement air $\pi R^2 \rho \tau_1 \cdot V$ acquires the same speed at the shock front, which it subsequently loses behind the front. But we do not know the speed of the shock. Moreover, we do not see it; we only see the smoke area located just behind the shock. We can definitely say that the velocity of the smoke front and shock velocity is not lower (but higher) than the speed of the rocket. By substituting shock velocity in the law of conservation of linear momentum with the rocket speed, as an equation we obtain:

$$\pi R^2 \rho \tau_1 \cdot V^2 < F \cdot \tau$$

Time $\tau_1 = 0.8$ is obtained by counting the frames on which the cloud conceals the rocket.

The air density at the staging altitude of 67 km, as published by NASA, and at nearby altitudes of 65 and 60 km, is obtained by interpolation of the table data of a standard atmosphere. In contrast to [1], where the standard atmosphere models of the 1960s were used, here we use the modern model [6]. It does not give significant differences in this calculation. But methodologically it is more correct.

Table 2.

The radius of the leading edge of the cloud	The speed at altitudes of separation. Less(m/s)		
	H=60 km $\rho = 3.9 \cdot 10^{-4} \text{ kg/m}^3$	H=65 km $\rho = 1.9 \cdot 10^{-4} \text{ kg/m}^3$	H=67 km $\rho = 1.7 \cdot 10^{-4} \text{ kg/m}^3$
50 m	920	1320	1370
60 m	770	1100	1160
70 m	660	940	1000

From the above table we can obtain the upper limits of the rocket velocity for different altitudes and different radii of the leading edge.

Measurements in [1] evaluated the front radius as 61 ± 3 m.

Below are some details regarding the method of radius measuring.

The radius of the leading edge was obtained in [1] by direct measurement of transverse to trajectory size of the more or less flat part of the cloud at the front. This measurement is compared with the rocket length; taking into account the distortion that occurs due to the rocket not being parallel to the film frame. In [1] the length of the visible part of the rocket was assumed to be 100m (i.e., not taking into account the length of the nose needle of 10m).

More details on the distortion factor:

A half cone angle of oblique shock on a good rocket photograph at the end of separation is clearly visible and well measured as 26 degrees. And the same angle in the film footage is 66 degrees.

However, in the photograph it is also distorted but visibly slighter. The nose cone angle, which is 19 degrees in the photo at the launch pad, does not increase to more than 20 degrees in the photograph. Taking into account all the errors, we can estimate that the actual shock angle is not less than 22.5 degrees. Then the scale in radial direction to the trajectory should be 1.57 times less than scale along the trajectory.

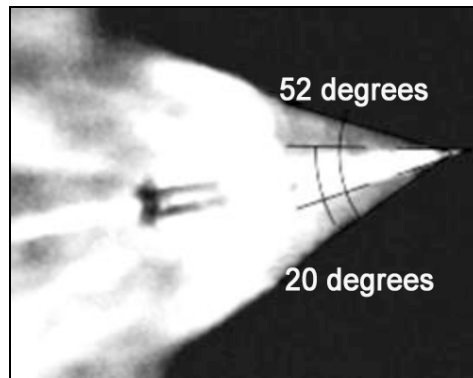


Image 3. Nose cone angle and the full oblique shock angle at the Apollo 11 first stage separation [5]. Accounting for distortions in the range of 10-15% gives a half angle of oblique shock of not less than 22.5 degrees.

Now the estimate of the front radius in [1] can be adjusted.

In many film frames the cloud front has a flat look which is fairly convenient for size measurement. But between the frames 201 and 205 (the second to last line of frames in Image 2) the front is sharpened. And measurements, carried out in [1] gave a small diameter at the front. But solid fuel engines have irregular thrust and uneven pressure in the combustion chamber. They first grow and then subside. Frames 201-205 correspond to the development of smoke clouds when retro-rockets are at their maximum performance. Decreasing diameter of the front at this time is a physical absurdity. We must interpret the situation in frames 201-205 as another breakthrough of a plane front with a diameter, say, the same as in frame 199 by retro-rocket gases, and corresponding cloud sharpening. For these frames we cannot consider the front "shrunk". Statistically for the average diameter it should remain at least at the level of frame 199. This clarification raises the estimate of the average radius of the front to 62.6m. The dispersion (variance about the mean) is somewhat reduced.

The next improvement is purely methodological in nature. We should not calculate an average radius but an average cross section. The radius calculated from the mean square of the measured dimensions was 63m.

The difference with $R = 61\text{m}$ in [1] is not significant. But we have ruled out methodological and physical inaccuracies.

A much greater change in our estimate occurs if we assume that we can see the rocket with the needle. In this case, the front radius increases by 10% and is very close to 70m. The speed estimate in this case drops to the level of $V < 1000\text{ m/s}$. But there is no serious ground for believing in visibility of the needle and we should not insist on this.

Further: three frames, numbers 189-191 were not taken into account in the evaluation of air momentum, on which the front is still small. But its speed in the direction of the rocket nose on average on these frames is 800 m/s higher than the rocket speed.

Three frames more 211-213 (Image 5) were not taken into consideration for the air momentum evaluation in which the leading edge of the cloud remains almost level with the rocket nose but had

already started decelerating. In any case, the shock front velocity is almost the same as the rocket velocity. And it captures and accelerates air molecules.

This is approximately 15% more time for the shock and air interaction which was underestimated in the evaluation. The speed estimate was thus overrated relative to the real speed by 5-7% and only at the expense of frames 211-213.

Based on these observations it can be stated that the evaluation $V < 1150$ m/s (at front radius of 60 m and altitude of 67 km) is reliable. The inequality is strong.

It should be immediately noted that with such a finite first-stage speed the rocket was not able to climb to a separation altitude of 67 km (obtained by direct computer calculation). It was only able to be at 61-63 km. That is why we have shown how speed estimation changes with altitude.

"Sharpening the front"

Despite the fact that an assumption of the front sharpness at the retro-rockets' maximum performance does not introduce any major changes in speed estimate, the question itself is interesting and barely described. In any event, the terminology used is unusual.

Nevertheless, the author has his own experimental data illustrating this point.

In studying the lowering of the optical rupture threshold for shock waves [4], shadow photographs illustrating the events were obtained. In the above Image 1-b, besides the main (primary) shock wave, represented by an arc of large radius there is a visible small curve inside the shock. The air behind the shock front has increased density and ionization, and is being heated by laser radiation. There is a local thermal explosion. The shock wave of a new explosion comes in the compressed hot air behind the primary shock, and compresses and heats it even further. And when it comes to the front of the primary shock, it breaks through the front. A rapidly forward-moving horn is formed. After its braking, the process is repeated at the head of a new front. Multiple breaches of the shock front arise.

The camera is not able to register the rapid initial motion of the shock; exposure (less than 15 nanoseconds) was barely enough when the front velocity dropped below 5 km/s. But a way of e-photo scanning showed speeds up to 100 km/s during rapid motion out of the "breach". The group of fronts had an average speed of development of 50 km/s; see Image 4b below.

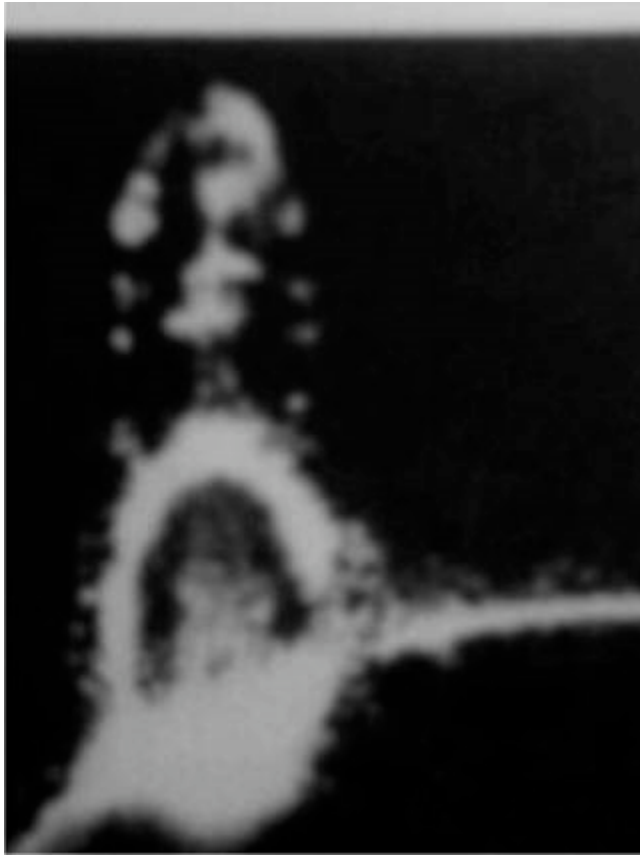


Image 4a

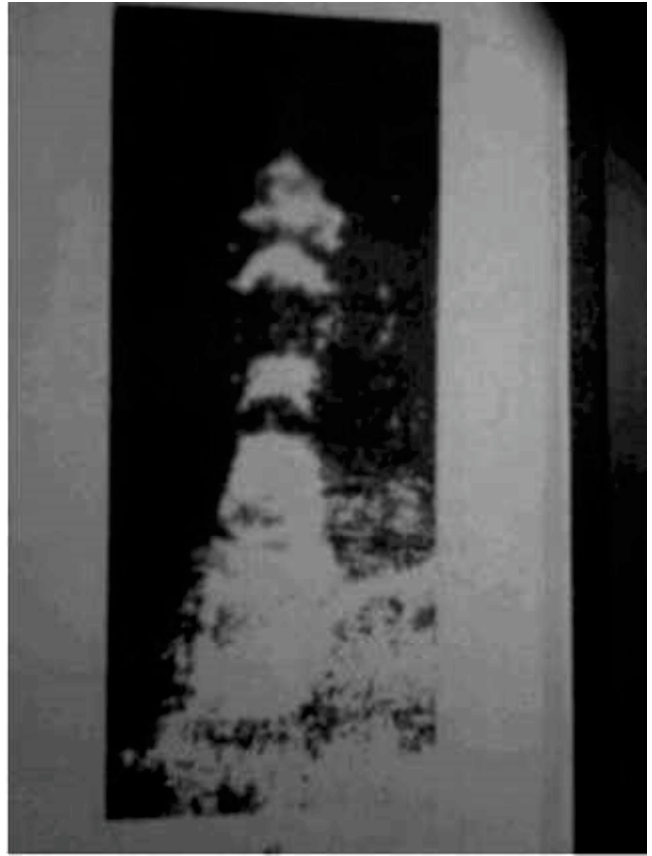


Image 4b

Image 4. Formation of successive front breaches of the primary shock.
 a) Observation of two successive shock fronts ahead of the primary shock;
 b) The number of secondary fronts reaches six.

The case of the Saturn V is quite similar. The source of energy for a local shock front breach and sharp front horn formation is not a laser, but a jet of increasingly dense and high-speed gases, as the burning solid propellant gains strength. It catches up with the front and further squeezes it. The air, compressed by the existing shock, resides aside of this local region of strong compression and high temperature. The actual jet, squeezing the front, is at the rear. The undisturbed air, showing the least resistance is at the front. A new explosion rushes forward.

The initial shock does not disappear. It keeps moving as the “information” from a new high-power disturbance has not yet propagated. But a rapidly-moving horn is growing ahead of this shock. Actually this is the front sharpening.

Speed estimate based on exhaust smoke cloud lag

In [1] an episode of the lagging smoke cloud after the retro-rockets cease firing was considered.

It was shown that the velocity lag of the fragment of the cloud front from the rocket nose cone increases and stabilizes at about 1300 m/s.

We previously found that velocity lag equals the speed of the rocket from the consideration that the speed of the cloud decreases relative to the stationary air to zero.

In this case, an error in understanding the physical depiction of events was introduced. Behind the shock, air is greatly warmed up. If, for example, a shock is moving through the air at room

temperature at a speed of 1000 m/s, its temperature after the front passage increases to 750 K. Behind the front it continues to rise due to energy thermalization of the directed motion of the molecules.

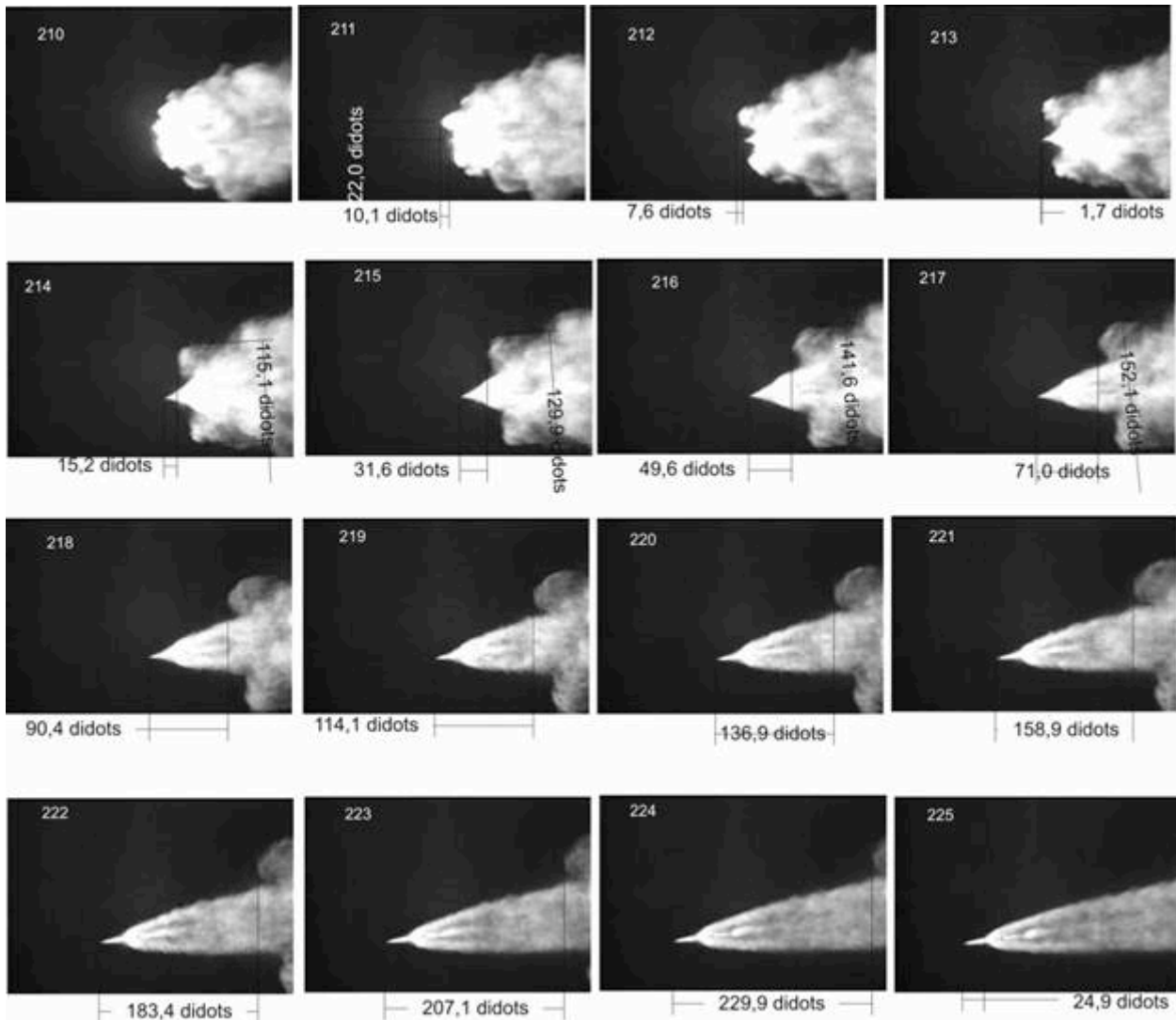


Image 5. The lagging aerosol cloud of solid fuel retro-rockets' residues after completion of their work, frames 210-225 of the film footage. In [1] an estimate of the rocket velocity $V < 1200-1600$ m/s was obtained based on this group of frames.

Heated to high temperature, air cools due to radiation and radial expansion. As a result, a region of low pressure is formed behind the rocket. The smoke cloud does not just stop in still air, it accelerates rearwards. The maximum possible acceleration speed to the vacuum in the theory [9] is:

$$u_{\max} = \frac{2}{\gamma - 1} c_{\text{H}}$$

Where C_{H} is the speed of sound in a stationary gas;

γ being the ratio of the specific heat capacity for constant pressure and the specific heat capacity for a constant volume (the adiabatic index). For air $\gamma = 1.4$.

If the speed of sound in still air is 300 m/s, the maximum possible acceleration speed for air flowing into the vacuum reaches 1500 m/s. The collapse of smoke behind the rocket is very clearly visible in frame 193 of the footage [5]; see Image 6.



Image 6. Frame 193. A rapid narrowing of the exhaust fumes behind the rocket.

It is clear that there is no vacuum behind the cloud front.

In this case, the speed of flowing into the low pressure region is as follows:

$$u \sim (c_1 - c_2) \cdot \frac{2}{\gamma - 1}$$

As already mentioned, at a shock velocity of 1000 m/s the air at the front is heated to 750K. Let us estimate the speed of sound in air at such a temperature:

$$c(750) \sim c(273) \cdot \sqrt{\frac{750}{273}} = 330 \cdot \sqrt{2.75} \sim 550 \text{ m/s}$$

If, for example, the air in the area behind the rocket has cooled to 500K (where the speed of sound is about 450 m/s), the speed at which the still hot air of the stopped front starts returning to the cooled space is close to 500 m/s. The cooling limit of a low pressure space behind the rocket is the ambient temperature. At the separation altitude it is just over 200K (where the speed of sound is about 300 m/s).

The uncertainty of the cloud's rearward velocity can be quite significant. We can only say that the rocket speed is below 1300 m/s (or any other speed obtained with the adjustments for cloud geometry and observation conditions). And it may be much lower.

An attempt to estimate the speed based on the lug (protuberance) in the smoke plume rather than based on the distant (from the rocket) cloud fragment turns out to be very interesting. This assessment was carried out by Alexander Reshnyak on the Internet discussion forum of the study [1].

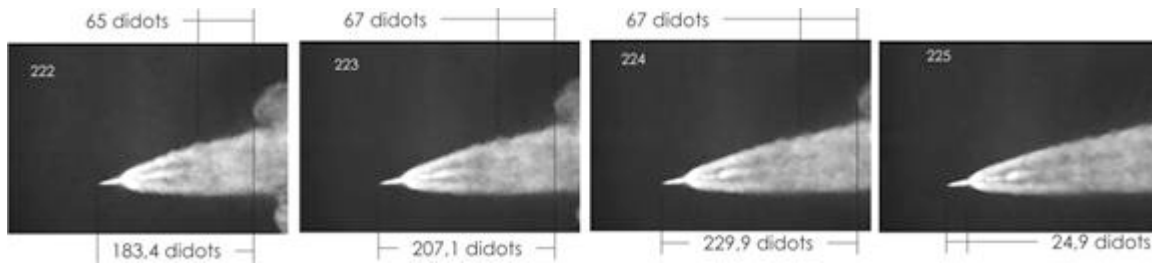


Image 7. Protuberance in the exhaust plume lagging behind the rocket nose cone.

The protuberance in the exhaust plume extends rearwards at the same distance from the front of the remote smoke cloud fragment. But this very protuberance in the smoke plume originated from the initial firing of the second stage engines. i.e., it is moving in the second stage engines' exhaust jet. The velocity lag of this protuberance from the rocket nose cone consists of the rocket speed itself and the rearward speed of the jet gases relative to the still air. This velocity lag has to certainly exceed the speed of the rocket. But in our case (not ruling out that just by a chance) this coincided with the velocity lag of the cloud fragment, remote from the flight axis, from the rocket nose cone.

$V < 1300-1450$ m/s (depending on the assumption as to whether we can see the rocket with or without its needle).

Interestingly, this estimate, in contrast to the estimates in [1] contains almost no questionable items regarding influence of the smoke's radial expansion or distortions associated with the observation geometry. And (being deliberately exaggerated) in this case it coincided with the measurements in [1].

Note how the logic has changed in comparison with [1]. Whereas in [1] the speed obtained by the exhaust plume lag was considered close to the correct value in both the upper- and lower-bound estimates, now we make an upper-bound estimate only!

We have identified a factor, due to which our estimate was certainly inflated. This dramatically simplifies coordination of the estimate obtained by the other methods.

And in terms of analysis of the feasibility/unfeasibility of this lunar mission; we do not need anything further.

Nevertheless, some questions from the previous paper must be discussed.

The effect of radial motion

The biggest question in the assessment was raised with regard to the evaluation of the position of the smoke cloud, which was far from the trajectory axis, and the possible influence of the cloud radial expansion on the result.

It is quite clear that the radial expansion of the cloud has the greatest effect when the velocity of radial expansion is at its maximum. But these are the very same frames where the cloud has the highest trajectory velocity comparable to the rocket speed.

In fact, an assessment of the rocket speed was carried out using the last group of frames when the lag velocity had stabilized. What happened to the radial development of the cloud? See Image 8.

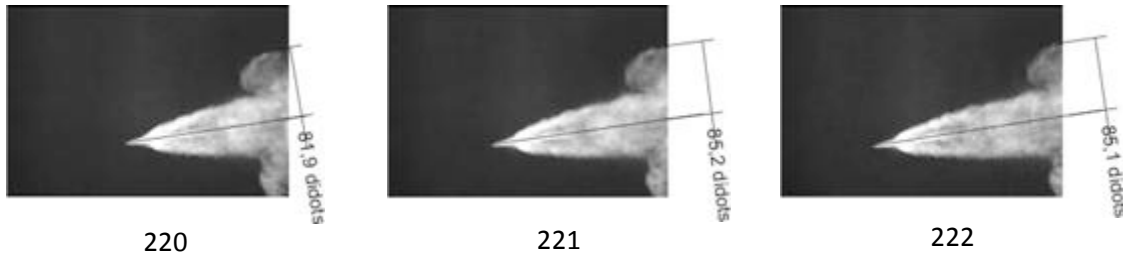


Image 8. Radial development of the cloud fragment in frames 220-222.

If the cloud is still growing between frames 220 and 221 (at a speed of about 240 m/s), then between frames 221 and 222 the radial cloud expansion is no longer registered. The radial velocity decreases to zero. We can no longer see further speed progress: the topmost point of the cloud fragment disappears out of the frame and other measurement benchmarks are not very accurate. Based on the above representation regarding the subsequent collapse of the smoke cloud, we can assume that on successive frames 223 and 224 the radial motion will acquire a negative speed to the trajectory axis. And it will introduce a speed overestimate error rather than a speed underestimate.

But even a zero radial velocity of the cloud, recorded in frames 221-222 provides a reasonable basis to conclude that no serious underestimation of the velocity was made in the previous paper due to cloud radial expansion.

In addition to the radial and axial velocities, there is one more – the tangential component perpendicular to the axial and radial ones. But it does not have any other origins besides turbulent evolutions with the speed of about the same as the radial velocity or less. From this point of view there may not be any further questions regarding the estimates in [1].

Oblique shock and boundary layer

In addition to normal shock examined in the previous information inset, supersonic movement of streamline bodies causes so-called oblique shock. The theory of oblique shock is well described in the classic work by Loitsiansky [8]. The incident stream is deflected by the surface.

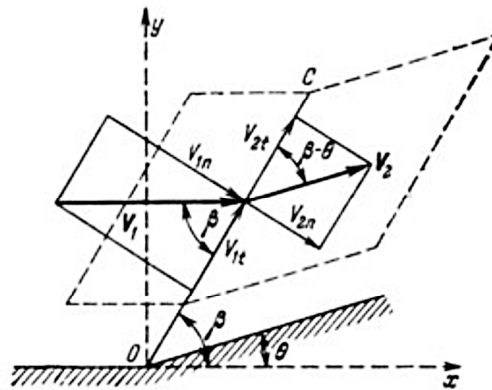


Figure 1. Diagram of angles and velocity components in the incident upstream deflected by a wedge forming an oblique shock [5].

In this case, the tangential-to-the-shock velocity component remains unchanged, and a normal one is reduced. (The line OC in Figure 1 is a shock cross section, the speed is projected to this axis and to its perpendicular; the components are labelled by indices: t – tangential, along OC or parallel to the shock front, n – normal, perpendicular to OC , i.e. to the shock front).

The air after the shock is compressed. The necessity of a shock appearance and its angle is determined primarily by the flow continuity assumption.

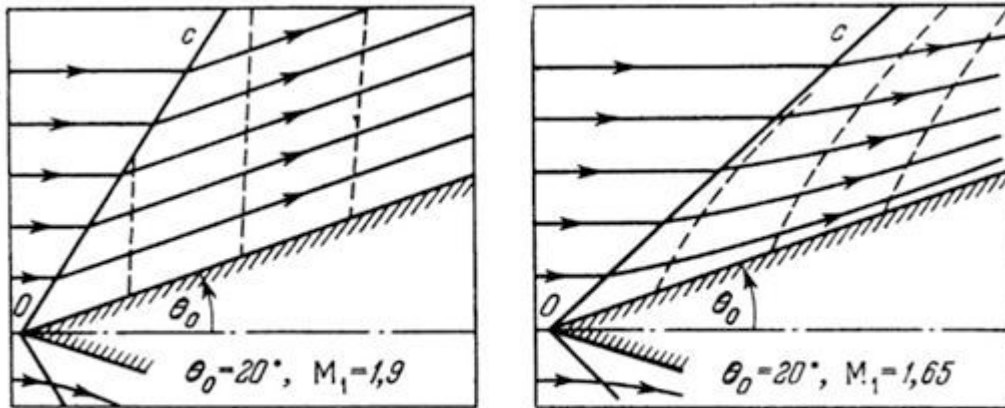


Figure 2. Comparison of a wedge (left) and a cone (right) streamlining [8].

The angles of a wedge (corner angle) and a cone angle θ_0 are equal. Airflow runs on the wedge with $M = 1.9$, and on the cone with $M = 1.65$. The forming oblique shock angle is noticeably larger on the wedge than on the cone.

For an oblique shock on a wedge there are elaborate diagrams and charts allowing determination of speeds by known angles, expressed in Mach numbers. A Mach number M is a ratio of flow or a body speed in relation to the speed of sound in a medium.

In [1] a diagram and a chart from [8] were presented.

Streamlining around a cone requires more complicated mathematical theory. But there is a very important feature of streamlining around a cone compared to a wedge. With equal corner angle and cone angle, the shock angle is less in the case of cone streamlining. This is well illustrated in the figure in [8], which is provided again here.

The next important concept is the boundary layer. Near the surface of a body moving in air a layer is formed as if it has "stuck" to the surface. If a body is not sufficiently smooth, small surface irregularities are smoothed out, "plastered up" by this boundary layer. This is clearly seen in Image 9.



Image 9. Interferogram of a streamlined plane with a notch [3].

Between the tip and the edge of the notch there is a region of surface smoothing in sluggish air. Bent bands of the interferogram show that a straight borderline is formed between a boundary layer and the streamlining flow. An oblique shock behaves as if the stream is flowing over a conventional wedge.

Speed evaluation based on the oblique shock angle

As stated previously, there is a good quality Apollo 11 picture (Image 3) where a shock wave is clearly visible with a half-angle of 26 degrees. Accounting for any angle distortion, as the rocket trajectory is not quite parallel to the photograph, we suggest that this angle is in reality no less than 22.5 degrees. All this corresponds to the arguments in [1]. But now we have to correct the error that appeared there. In [1] we assumed that the shock cone was formed on the nose cone of the third stage with half-angle 9.5 degrees. However, the shock length along the rocket is much greater than the length of the nose cone. And, as we have shown in the information inset, the oblique shock angle is determined by flow continuity assumption. If the shock angle was defined by a nose cone, then after reaching the cylindrical section of the third stage, it would turn down.

The considerable length of the shock straight edge is explained by taking into account the boundary layer effect. Between the nose cone of the Apollo 11 Command and Service Module and the beginning of the cylindrical second stage a "plastered"-by-a-boundary layer irregularity is formed, working in the stream as a cylinder with a half-angle of six degrees.

If a wedge with such an angle moves through air, the speed corresponding to the shock angle of 22.5 degrees, would reach 3.2 M. The rocket was not streamlined as a wedge but as a cone, and at such a shock angle its velocity would be lower. But for upper-bound speed estimate a wedge approximation is quite sufficient.

At the NASA stated separation altitude of 67 km the temperature is obtained by a linear interpolation of the standard atmosphere table data. It is 231K and the speed of sound of 304 m/s corresponds to this temperature. Therefore the rocket velocity was less than 970 m/s.

According to our estimates, a rocket with such a low final speed cannot reach a height of 67 km. The actual separation altitude seems to be a few kilometers different from the declared data. And there is a noticeable rise in temperature below and accordingly a change in the speed of sound.

The speed estimate of Mach 3.2 for the altitude, for example, 60 km corresponds to 1030 m/s. But this is a wedge estimate. The rocket speed was lower.

Any inaccuracy of this assessment can be related to the difference of atmosphere in the zone of the active part of the trajectory from the average standard atmosphere. The old atmosphere model was used in [1], in which at altitude of 65 km the temperature was 319K and the speed of sound close to

360 m/s. At an altitude of 60 km it increases to 375 m/s. We take these latest numbers as an upper limit of the possible speed of sound at the separation altitude. Then $V < 1150-1200 \text{ m/s}$, which is perfectly correlated with the speed estimate obtained from the law of conservation of linear momentum.

Logic clarification

Now let us take a closer look at the logic of why we had to abandon the use of a nose cone as giving configuration to the shock wave, and talk about another cone.

Proof by contradiction.

Assume that the position of the oblique shock at the beginning is determined by the third stage nose cone with half-angle 9.5 degrees.

If it had not been just a few meters long, but was much longer (by tens of meters), then at a constant geometry determining surface streamlining, the shock cone would have remained exactly the same.

In reality there is a transition to the cylindrical section behind the third stage nose cone. Since we see the straight boundary of the shock cone in the photograph, it means that:

- 1) Either a large volume cavity is formed, where air does not want to enter; as it is physically impossible
- or
- 2) This region is not available for streamlining air jets due to air plug presence. Such an air plug is known in science; it is a still (relative to a moving body) boundary layer filling any surface irregularities.

But if the boundary of the third stage nose cone is imagined to extend by several tens of meters, then this line extends into space, and does not end up on any physical boundaries associated with the rocket. It is clear that the boundary layer cannot exist far enough from the rocket body. Hence, our assumption that the shock is determined by the angle of the third stage nose cone, stated in the first article, is wrong. An extended boundary layer cannot exist with such an angle.



Image 10. Assumed position of the boundary layer edge in supersonic flow of the rocket body. The line connecting the nose cone with the Command and Service Module beneath it and the beginning of cylindrical section of the second stage forms an angle of six degrees with the axis of rocket.

Consequently, there is another surface to boundary layer, within which the third stage nose cone is located. The only natural border of the boundary layer is placed under the conical surface connecting the Command and Service Module nose cone edge with beginning of the second stage cylindrical part. See Image 10 above.

The third stage nose cone is entirely under this surface, touching it in the joint of the adapter and the cylindrical section of the third stage.

Discussion of the Results

A series of rocket speed estimates was carried out. The first calculation measured the radius of the exhaust smoke cloud and was based on the NASA data of the retro-rockets' parameters and the

stages' separation altitude. The assessments obtained reliably conclude that any credibility of the NASA data is totally out of the question.

Nonetheless, let us look at the structure of the speed assessment in the first case. The upper speed limit is proportional to the square root of the product of the solid fuel engine's thrust and the time of their work, i.e. momentum, which the retro-rockets delivered to the first stage. To obtain the "correct" (according to NASA) rocket velocity of about 2350-2400 m/s a more than four times increase of corresponding momentum is required. But according to the documentation, the momentum was delivered to an object weighing 100 tons. The corresponding calculated velocity lag of the first stage should then be about 80 m/s. But, as noted in the first information inset, 13 seconds after the start of separation of the first stage the lag was about 180m.

Either the NASA data on the solid propellant rocket performance is close to the truth, or the mass of the first stage is four times greater – not 100 tons, but 400 tons. What then can be propelled to the Moon? That is the important question. Conversely, reducing the thrust against the declared data requires an even lower rocket speed.

Any assumption that could seriously affect the speed estimate based on the cloud size is refuted by the photographic evidence of successive seconds as discussed in the previous paragraph.

The separation altitude data is somewhat more complicated. Several kilometers above the stated altitude of 67 km the air density becomes so low, that at NASA's stated speed, the observed cloud diameter is entirely correct.

However, it is important to emphasize that we have not carried out a single estimate. We have an estimate based on the shock cone angle. A temperature decrease was observed at altitudes of 65-80 km, which corresponds to the speed of sound decrease. But the shock angle requires a speed not more than 3.2 Mach.

An achieved speed of no more than 1150-1200 m/s is the maximum possible speed for the entire range of altitudes from zero to 90 km for which there is an atmospheric aerodynamic streamlining and where shocks are formed.

Conclusion

The possible upper speed estimate of the Saturn V booster at the separation point has been refined in this second paper.

The Apollo 11 Saturn V booster speed was strictly **slower** than 1150-1200 m/s.

Only the upper-bound speed estimate is significant. We did not carry out a low-bound speed estimate in reaching a conclusion concerning the possibility/impossibility of the mission. Our assumption that NASA's data on the separation altitude is incorrect and misleading cannot change the overall conclusion reached here regarding the impossibility of the Apollo 11 mission.

At the same time the method of a speed estimate via the cloud lag used in [1], was analyzed. A physical picture of events was refined in which a significant source of errors in determining the rocket speed is the collapse of the low-density cooled area behind the rocket. The exhaust cloud velocity lag was higher than the rocket speed by the value of the cloud's rearward motion towards the low pressure behind the rocket. The method used was appropriate for demonstration purposes, but delivered a considerable velocity overestimation.

At the same time it was shown that the radial evolution of the cloud did not cause any significant speed underestimation in [1].

The resulting refined velocity is

$$V < 1150-1200 \text{ m/s}$$

This result clearly indicates the impossibility of delivering a return Apollo mission to the Moon.

At specified conditions in the Apollo program every meter per second of delta-v corresponded to 15 kg of payload to the Moon [10]. In the same source data, the speed at the separation point of 2750 m/s was provided. Given that the Earth's rotation speed is about 400 m/s, more than half of the rocket velocity deficit relative to the air was derived, compared to the required speed that was necessary to satisfy the Apollo 11 flight plan.

In the first approximation with a velocity deficit of about 1150-1200 m/s, it was not possible to put a deficit of 17 000-18 000 kg into lunar orbit. NASA stated that the mission placed about 46 tons into lunar orbit – 28 tons of which had to be the Apollo craft itself. Since all our speed estimates are upper-bound, the booster payload deficit may have been even higher.

Given these obtained estimates, all arguments over what could have been achieved during the Apollo program should take into account that not more than 28 tons, including the Apollo 11 craft itself, out of 46 tons (as stated by NASA) could have been placed into lunar orbit.

References

1. С.Г. Покровский. Попасть на Луну американцы не могли // Актуальные проблемы современной науки. 2007. № 5, с.152-166. (S. G. Pokrovsky. Investigation into the Saturn V velocity and its ability to place the stated payload into lunar orbit).
2. Г.Райхенбах. Ударные волны в газах. В сб. Физика быстропротекающих процессов. Перевод под ред. Н.А.Златина. III том. – М.: Издательство «Мир», 1971- 360 с.
3. Х.Эртель. Измерения в гиперзвуковых ударных трубах. В сб. Физика быстропротекающих процессов. Перевод под ред. Н.А.Златина. III том. – М.: Издательство «Мир», 1971- 360 с.
4. Покровский С.Г. Лазерная обработка тонких металлических пленок для задач полиграфического машиностроения. Дисс. канд. тех. наук. М.: ИМЕТ, 1998, -169 с.
5. http://spaceflight.nasa.gov/gallery/video/apollo/apollo11/mpg/apollo11_launchclip03.mpg
6. <http://www.krugosvet.ru/articles/118/1011820/1011820a1.htm>
7. Michael Light. *Full Moon*. London: Jonathan Cape – 1999. – All photographs courtesy National Aeronautics and Space Administration
8. Лойцянский Л.Г. Механика жидкости и газа: Учеб. Для вузов. – 7-е изд., испр. – М. Дрофа, 2003, - 840 с.
9. Орленко Л.П. Физика взрыва и удара: Учебное пособие для вузов.- М.: ФИЗМАТЛИТ, 2006. – 304 с.
10. И.И.Шунейко. Пилотируемые полеты на Луну, конструкция и характеристики SATURN V APOLLO// Итоги науки и техники. Сер. Ракетостроение. М. 1973.

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